Collective pinning of imperfect vortex lattices by material line defects in extreme type-II superconductors

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Abstract

The critical current density shown by a superconductor at the extreme type-II limit is predicted to follow a $1/\sqrt{B}$ power law with external magnetic field B if the vortex lattice is weakly pinned by material line defects. It acquires an additional inverse dependence with thickness along the line direction once pinning of the interstitial vortex lines by material point defects is included. Moderate quantitative agreement with the critical current density shown by second-generation wires of high-temperature superconductors in kG magnetic fields is achieved at liquid-nitrogen temperature.

Thin films of the high-temperature superconductor $YBa_2Cu_3O_{7-\delta}$ (YBCO) grown by pulsed laser deposition (PLD) on a substrate that serves to align the crystalline axes now routinely achieve critical currents that reach a substantial fraction of the maximum depairing current[1][2][3][4]. Such films also typically show a peak in the critical current as a function of the orientation of external magnetic field near the crystallographic c axis, perpendicular to the film (copper-oxygen) plane[5]. It is well known that the Abrikosov vortex lattice that exists in external magnetic field shows no critical current if it is not pinned down by material defects[6][7]. The c-axis peak in the critical current shown by films of PLD-YBCO therefore indicates that some fraction of the material defects that pin the vortex lattice must be correlated along the c axis as well. Careful studies of the microstructure in such YBCO films find that the correlated pinning centers in question are in fact lines of dislocations that appear naturally in the PLD growth process[2][8]. The pinning centers notably arrange themselves in a manner that resembles a snapshot of a two-dimensional (2D) liquid, as opposed to a snapshot of a 2D gas.

Below, we explore the theoretical consequences of the proposal that the vortex lattice induced by an external magnetic field oriented near the c axis in films of PLD-YBCO is in a thermodynamic Bose glass state characterized by an infinite tilt modulus[9]. We specifically focus on the high-field regime at the extreme type-II limit, in which case only a fraction of the vortex lines are localized at the dislocations that thread the film along the c axis[10][11][12], and in which case the pinning of the vortex lattice is collective[6][13][14]. The theory predicts a critical current density along the film that approximately follows a $1/\sqrt{B}$ power law as a function of magnetic field B in the collective pinning regime. It successfully accounts for the critical current density obtained in c-axis field from films of PLD-YBCO that are microns thick, at liquid nitrogen temperature. It fails, however, for much thinner films at lower temperature. We demonstrate that this failure can be corrected by including the effect of point pins on the interstitial vortex lines that lie in between the correlated pins. They contribute an inverse dependence on film thickness to the the critical current density in magnetic field oriented near the c axis.

Consider again the film of YBCO just described in large enough external magnetic field applied along the c axis so that magnetic fluxlines overlap considerably. The vortex lines experience long range repulsive interactions in such case, which favors their arrangement into a triangular lattice. The natural material line defects that also run parallel to the c

axis act to pin down some fraction of the vortex lines, on the other hand. This frustrates the formation of a vortex lattice. The resulting thermodynamic state is a Bose glass[9] that displays a divergent tilt modulus because of pinning by the material line defects[10][11][12].

The vortex lattice described above is effectively 2D due to the strong orientation along the c axis. Theoretical and numerical calculations then indicate that a net concentration of straight lines of unbound dislocations will be quenched into the triangular vortex lattice at zero temperature[14][15][16]. Monte Carlo simulations of the frustrated XY model further show that the dislocations in the vortex lattice appear either unbound or bound-up into neutral pairs in the extreme type-II limit[11]. In particular, the lines of dislocations do *not* arrange themselves into low-angle grain boundaries (cf. refs [17] and [18]).

We shall now compute the critical current density that is expected along the film direction for the hexatic Bose glass described above[11][12]. The vortex lattice is pinned collectively[6] when the number of vortex lines localized at material line defects is small compared to the total number of vortex lines. This shall be assumed throughout. The positional coherence of the vortex lattice along the c axis implies 2D collective pinning in particular[6][13], with an infinite Larkin length in that direction[12]: $L_c \rightarrow \infty$. Next, we shall assume that the critical current density is limited by *plastic creep* of the vortex lattice due to slip of the quenched-in lines of dislocations along their respective glide planes[19]. The Larkin length in the direction transverse to the magnetic field, R_c, is then obtained by minimizing the sum of the elastic energy cost and the gain in pinning energy due to the translation of a Larkin domain by an elementary Burgers vector of the triangular vortex lattice, $b = a_{\triangle}$. This yields the estimate[6][13]

$$R_c^{-2} = C_0^2 n_{\rm p} (f_{\rm p}/c_{66} b)^2, \tag{1}$$

for the density of Larkin domains, where $n_{\rm p}$ denotes the density of pinned vortex lines, where $f_{\rm p}$ denotes the maximum pinning force per unit length along a material line defect, and where c_{66} denotes the elastic shear modulus. The prefactor C_0 above is of order unity. On the other hand, because the Larkin correlation volume corresponds to the largest bundle of vortex lines that exhibits a purely elastic response[6], it is natural to identify the average separation between lines of dislocations that are quenched into the vortex lattice with the transverse Larkin length, R_c. A similar minimization of the sum of the elastic energy and the pinning energy then yields the same form (1) for the density of Larkin domains in such case[14], but with a prefactor $C_0 \cong \pi / \ln(R_c/a'_{\rm df})^2$. Here $a'_{\rm df}$ is of order the core diameter

of a dislocation in the vortex lattice. Finally, the statistical nature of 2D collective pinning requires many pinned vortex lines per Larkin domain[6][7]: $n_{\rm p} > R_c^{-2}$. Comparison with relation (1) then yields the threshold in magnetic field

$$B_{\rm cp} = C_0^2(\sqrt{3}/2)(4f_{\rm p}/\varepsilon_0)^2\Phi_0, \tag{2}$$

above which 2D collective pinning holds. The estimate $c_{66} = (\Phi_0/8\pi\lambda_L)^2 n_{\rm vx}$ for the shear modulus of the vortex lattice[20] has been used here, where $n_{\rm vx}$ denotes the density of vortex lines, where λ_L denotes the London penetration depth, and where Φ_0 denotes the flux quantum. Above, $\varepsilon_0 = (\Phi_0/4\pi\lambda_L)^2$ is the maximum tension of a fluxline in the superconductor.

The Lorentz force balances the pinning force in the critical state following $j_c B/c = (R_c^{-2}\cdot n_{\rm p})^{1/2} f_{\rm p}$ when 2D collective pinning holds[6]. Here j_c denotes the critical current density along the film, which is perpendicular to the magnetic field B aligned parallel to the material line defects. Substitution of the result (1) for the density of Larkin domains reduces this balance to

$$j_c B/c = C_0(c_{66}b)^{-1} n_{\rm p} f_{\rm p}^2. \tag{3}$$

The critical current density can then be determined from the above condition for mechanical equilibrium once the density of pinned vortex lines, $n_{\rm p}$, is known. Non-interacting vortex lines, $c_{66}/f_{\rm p} \to 0$, yields the upper bound for $n_{\rm p}$. The profile of $n_{\rm p}$ versus magnetic field in this case is clearly just the upper pair of dashed lines that join at the density of material line defects, n_ϕ, shown in Fig. 1. Infinitely weak correlated pins, $c_{66}/f_{\rm p} \to \infty$, yields the lower bound for $n_{\rm p}$, on the other hand. It corresponds to the limit of an infinitely rigid vortex lattice. The profile of $n_{\rm p}$ versus magnetic field in that case follows the lower dashed line shown in Fig. 1, where $r_{\rm p}$ denotes the effective pinning radius of the material line defects. It is obtained by observing that $n_{\rm p}/n_\phi$ coincides with the probability that a given pinning center trap a vortex line, $p = \pi r_{\rm p}^2 \cdot n_{\rm vx}$. (See Fig. 2 and ref. [21].) This coincidence is true for *any* density of pinned vortex lines, $n_{\rm p}$, as long as the material line defects do not crowd together: $\pi r_{\rm p}^2 \cdot n_\phi \ll 1$. Such is the case with the lines of dislocations that thread films of PLD-YBCO, which display an effective hard-core repulsion[2][8].

The fact that the vortex lattice is well ordered within a Larkin domain (see Fig. 2) indicates that the direct scaling of $n_{\rm p}$ with B obtained above in the limit of an infinitely rigid vortex lattice persists in the regime of weak correlated pinning more generally. This in fact can be demonstrated by first observing that the relative statistical error in the number of pinned

vortex lines at $f_{\mathrm{p}} \rightarrow 0$ obeys the law of large numbers: $\Delta N_{\mathrm{p}}/\bar{N}_{\mathrm{p}} = f(\pi r_{\mathrm{p}}^2 \cdot n_{\mathrm{vx}})/\sqrt{n_\phi R_c^2}$, where N_{p} denotes the number of pinned vortex lines inside of a Larkin domain, and where $f(p) = \sqrt{(1-p)/p}$ sets the fluctuation scale specifically for the binomial probability distribution[22]. Second, adapting the statistical principle of Larkin-Ovchinnikov[6] to the present case yields the new rule that the break-up of the pristine vortex lattice into Larkin domains of dimensions $R_c \times R_c$ serves to increase the number of pinned vortex lines with respect to the most probable number, $\bar{N}_{\mathrm{p}} = p \cdot n_\phi R_c^2$ at $f_{\mathrm{p}} \rightarrow 0$, by a number of order the statistical error, ΔN_{p}:

$$\ln\left(\frac{n_{\mathrm{p}}/n_\phi}{\pi r_{\mathrm{p}}^2 \cdot n_{\mathrm{vx}}}\right) = c_0 \frac{\Delta N_{\mathrm{p}}}{\bar{N}_{\mathrm{p}}}. \tag{4}$$

Here c_0 is a numerical constant of order unity. The density of pinned vortex lines therefore scales nearly directly with magnetic field at small relative errors, $\Delta N_{\mathrm{p}}/\bar{N}_{\mathrm{p}} \ll 1$. This requires collective pinning, $\bar{N}_{\mathrm{p}} \gg 1$. Substitution into Eq. (1) yields a density of Larkin domains, $R_c^{-2} \cong (\sqrt{3}\pi/2) C_0^2 (4 f_{\mathrm{p}} r_{\mathrm{p}}/\varepsilon_0)^2 n_\phi$, that depends only weakly on the magnetic field in such case. Finally, the function $f(p)$ specific to the binomial probability distribution can be approximated by $(2/\sqrt{3}) \ln[(3\sqrt{e}/4)/p]$ for probabilities p inside of the range $[0.01, 0.98]$ (cf. ref. [21]). Substituting that approximation into Eq. (4) then yields the scaling result

$$n_{\mathrm{p}}/n_\phi = (B/B_{\phi 2})^{d_{\mathrm{p}}/2} \tag{5}$$

for the density of pinned vortex lines as a function of magnetic field oriented parallel to the material line defects, where $d_{\mathrm{p}}/2 = 1 - (c_1/\sqrt{n_\phi R_c^2})$ gives one-half the effective scaling dimension, and where $B_{\phi 2} = c_2 \Phi_0/\pi r_{\mathrm{p}}^2$ gives the saturation field. Here, we define $c_1 = 2c_0/\sqrt{3}$ and $c_2 = (3\sqrt{e}/4)^{-[(2/d_{\mathrm{p}})-1]}$. The last constant is notably less than unity. Fig. 1 depicts the field dependence of the scaling ansatz (5) schematically, where the correction to the scaling dimension is ignored.

Table I lists various limiting cases that obey the scaling ansatz (5) for the density of pinned vortex lines as a function of magnetic field, including the lower limit at weak pinning just treated. Substituting (5) into the condition (3) for the critical state then yields the power law

$$j_c(B)/j_c(B_0) = (B_0/B)^{\alpha_{\mathrm{cp}}} \tag{6}$$

for the critical current density as a function of magnetic field, with exponent

$$\alpha_{\mathrm{cp}} = (3 - d_{\mathrm{p}})/2. \tag{7}$$

The reference critical current density $j_c(B_0)$ at the geometric mean $B_0 = (B_{\rm cp}B_{\phi 2})^{1/2}$ of the range in magnetic field of the power law, $[B_{\rm cp}, B_{\phi 2}]$, determines the latter by the formula

$$\frac{B_{\phi 2}}{B_{\rm cp}} = \left(C_1^{-1}\cdot\frac{B_\phi}{\sqrt{B_0 H_{c2}}}\cdot\frac{j_0}{j_c(B_0)}\right)^{\left[1-\frac{1}{2}(\alpha_{\rm cp}-\frac{1}{2})\right]^{-1}}. \tag{8}$$

Here $j_0 = 4c\varepsilon_0/3\sqrt{3}\Phi_0\xi$ and $H_{c2} = \Phi_0/2\pi\xi^2$ are the the depairing current density and the upper critical field, respectively, each set by the coherence length ξ [7]. Also, the constant factor above is defined by $C_1 = (16\sqrt{\pi}/3^{5/4})C_0$. Study of Eqs. (1) and (4) again shows that $n_{\rm p}$ scales nearly directly with B in the collective pinning regime, $B > B_{\rm cp}$. By the previous Eqs. (5)-(7), we conclude that the critical current density then decays with magnetic field like $1/\sqrt{B}$ (see Table I).

Consider next the addition of a field of material point defects to the thin-film superconductor. Either the Bose glass state described above remains intact[9], in which case L_c remains infinite[12], or it will break up into smaller Larkin domains of finite thickness, $L_c < \infty$. In either case, consider henceforth films of thickness $\tau < L_c$. Collective pinning therefore remains 2D[6][13], and the *interstitial* vortex lines that remain free of the material line defects may be considered as rigid rods. The material point defects will pin the latter. Recall now the basic idea behind 2D collective pinning, which is that the critical pinning force per unit volume is given by the quotient of the variance of the net equilibrium force per unit length over a Larkin domain with the cross-sectional area R_c^2. The magnitude-square of this quotient is equal to

$$\overline{|\sum_{i\in{\rm LD}}\vec{f}(i)|^2}/R_c^4 = (n_{\rm p}f_{\rm p}^2 + n'_{\rm p}f'^2_{\rm p})/R_c^2. \tag{9}$$

Above, $\vec{f}$ denotes the force per unit length due either to a vortex line pinned by a material line defect or to an interstitial vortex line pinned by material point defects. These have respective root-mean-square values of $f_{\rm p}$ and $f'_{\rm p}$. Also, $n_{\rm p}$ and $n'_{\rm p}$ denote, respectively, the density of vortex lines pinned by material line defects and the density of interstitial vortex lines pinned by material point defects. Last, the overbar notation above denotes a bulk average achieved by rigid translations of a given Larkin domain (LD). The identity (9) is then due to the statistical independence of all of the pinning forces: $\overline{\sum'_{i,j\in{\rm LD}}\vec{f}(i)\cdot\vec{f}(j)} = 0$, where the prime over the summation symbol specifies that i and j denote distinct pinned vortex lines within a given Larkin domain. This last property is closely related to the null

average value of the net pinning force exerted on a Larkin domain when the vortex lattice is in mechanical equilibrium. In particular, $\sum \vec{f} = 0$ implies $\overline{\sum_{\rm LD} \vec{f}} = 0$. By Eq. (9), the additional source of pinning due to the interstitial vortex lines can then be accounted for by making the replacements

$$n_{\rm p} f_{\rm p}^2 \rightarrow n_{\rm p} f_{\rm p}^2 + n_{\rm p}' f_{\rm p}'^2 \quad \text{and} \quad j_c \rightarrow j_c + j_c' \tag{10}$$

in Eqs. (1) and (3) for the density of Larkin domains in the vortex lattice and for the critical state. Here, j_c' denotes the contribution by the point pins to the net critical current density, $J_c = j_c + j_c'$. The first replacement displayed by Eq. (10) indicates an *effective* density of vortex lines pinned by material line defects, $n_{\rm p} + (f_{\rm p}'^2/f_{\rm p}^2) n_{\rm p}'$. Notice that the latter correctly equals $n_{\rm p}$ when $f_{\rm p}' = 0$, and $n_{\rm p} + n_{\rm p}'$ when $f_{\rm p}' = f_{\rm p}$. Matching it to the modified result for the density of Larkin domains (1) then yields the old result (2) for the threshold field beyond which 2D collective-pinning holds. The previous Eqs. (3) and (8) then remain unchanged as long as j_c is understood to represent the contribution to the critical current density by the material line defects alone!

The condition (3) for the critical state that results from pinning of the vortex lattice by material line defects demonstrates that the critical current density j_c is a *bulk* quantity that is independent of the thickness of the thin-film superconductor along the axis of the correlated pins. This notably is not the case for the contribution by point pins to the critical current density, j_c'. In particular, the forces due to point pins add up statistically along a rigid interstitial vortex line if the film is much thicker than the average separation between such pins along the field direction, $\tau_{\rm p}'$. The effective pinning force per unit length experienced by an interstitial vortex line is then given by[13] $f_{\rm p}' = f_0'/(\tau_{\rm p}'\tau)^{1/2}$ at film thicknesses $\tau > \tau_{\rm p}'$, where f_0' denotes the maximum force exerted by a point pin. Also, all of the interstitial vortex lines are pinned by point defects if the film is thick enough: $n_{\rm p}' = n_{\rm vx} - n_{\rm p}$ at $\tau > \tau_{\rm p}'$. The relative contribution by point pins to the critical current density is then predicted to show an inverse dependence on film thickness, $j_c'/j_c = \tau_0/\tau$, that is set by the scale $\tau_0 = (n_{\rm vx}/n_{\rm p} - 1)(f_0'^2/f_{\rm p}^2 \tau_{\rm p}')$. The net critical current density, $J_c = j_c + j_c'$, then follows a pure $1/\sqrt{B}$ power law in the case that the density of vortex lines pinned by material line defects scales directly with magnetic field ($d_{\rm p} = 2$). More generally, we predict a linear dependence on film thickness for the net critical current per unit width following $K_c = (\tau_0 + \tau) j_c$, where $K_c = \tau J_c$ by definition.

The summation of pinning forces is *coherent*, on the other hand, at magnetic fields below the threshold (2) for 2D collective pinning: $j_c B/c = n_{\rm p} f_{\rm p}$ at $B < B_{\rm cp}$. Further, we have that $n_{\rm p}$ is approximately equal to $n_{\rm vx}$ at low magnetic fields compared to the accommodation scale[9] $B_{\phi 1} = (4\varepsilon_{\rm p}/\varepsilon_0)(\Phi_0 \cdot n_\phi)$, where $\varepsilon_{\rm p}$ denotes the depth of the correlated pinning potential per unit length. The above balance of forces then yields a critical current density that reaches a plateau

$$j_c(0+)/j_0 = (3\sqrt{3}/4)(f_{\rm p}\xi/\varepsilon_0) \tag{11}$$

in the zero-field limit[23]. Finally, the inequality $B_{\phi 1} < B_{\rm cp}$ shall be assumed throughout. It places the bound $B_\phi < (4\varepsilon_{\rm p}/\varepsilon_0)(\Phi_0/r_{\rm p}^2)$ on the matching field.

Thin films of YBCO grown by PLD on a substrate also show an extended regime in magnetic field oriented parallel to the c axis where the critical current obeys a power law (see Fig. 3 and refs. [1][2][3][4]). The films themselves are divided into columns of subgrains that run parallel to the crystallographic c axis. Etching of the film surface demonstrates that lines of dislocations also run parallel to the c axis in “trenches” that separate such growth islands[2][8]. There typically exists about one threading dislocation per growth island, each separated by a distance of about 110 nm [2][8]. The dimensionality of the columnar pins along a cross section then coincides with that of the subgrains, $d_{\rm p} = 2$. The critical current that is obtained from such films of YBCO in magnetic field B aligned along the c-axis typically obeys a $1/\sqrt{B}$ law, in particular[1][2][3][4]. This is consistent with a direct dependence, $n_{\rm p} \propto B$, of the density of pinned vortex lines on the magnetic field by the present theory (7) for 2D collective pinning. Such a dependence on magnetic field, in turn, is predicted approximately by Eq. (4) in the collective pinning regime. (See Table I).

Another point of comparison between theory and experiment is the actual magnitude of the critical current density. The maximum force per unit length exerted by a correlated pin, $f_{\rm p}$, can be obtained from j_c in self field via Eq. (11) [24]. Substituting it into the right-hand side of Eq. (2) then yields a prediction for the threshold field beyond which 2D collective pinning holds:

$$B_{\rm cp} = C_1^2 [j_c(0+)/j_0]^2 H_{c2}. \tag{12}$$

Values of self-field J_c measured in various films of PLD-YBCO[2][3][4] are listed in Table II, along side of the predicted threshold field, $B_{\rm cp}$. The London penetration depth is set to its value at zero temperature, $\lambda_L(0) = 150$ nm. A value for the superconducting coherence

length of $\xi(0) = 1.5$ nm then yields a depairing current density of $j_0(0) = 300$ MA/cm^2, as well as an upper-critical field of $H_{c2}(0) = 146$ T. Also, the prefactor C_0 on the right-hand side of Eq. (1) for the density of Larkin domains in the vortex lattice is set to unity. The predicted threshold magnetic field fails to lie below the geometric mean $\sqrt{B_1 B_2}$ of the power-law regime (see Fig. 3) only in the case of the relatively thin YBCO film[2][3][4]. This is likely due to the neglect of pinning by true grain boundaries[25].

The present theory for 2D collective pinning by material line defects also predicts the range of the power law in magnetic field (8) from knowledge of the critical current density at the logarithmic midpoint. (See Fig. 3, "$\times$".) Table II again lists recent measurements of the critical current density in films of PLD-YBCO at the logarithmic midpoint, along side of the predicted range. Only films that show an inverse power-law (6) in J_c versus B characterized by an exponent α in the vicinity of 1/2 are considered. Standard physical parameters for YBCO are again used, the prefactor C_0 is again set to unity, and ε_0 is again set to its value at zero temperature. Additionally, the effect of point pins is neglected ($j_c' = 0$). Comparison of the last two columns in Table II indicates that the present theory for 2D collective pinning gives a fair account of the critical current density in PLD films of YBCO superconductor that are microns thick. We believe that the failure of the theory in the case of the much thinner film is due to the neglect of the contribution by point pins.

In conclusion, 2D collective pinning of the vortex lattice by material line defects can account for the inverse-square-root power law obeyed by the critical current density in films of PLD-YBCO versus external magnetic field[1]. We also predict on this basis that the critical current per unit width is a linear function of film thickness, with a positive slope equal to the bulk critical current density, and with a negative intercept on the thickness axis, after extrapolation, due to pinning of interstitial vortex lines by material point defects.

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[1] B. Dam, J.M. Huijbregtse, F.C. Klaassen, R.C.F. van der Geest, G. Doornbos, J.H. Rector, A.M. Testa, S. Freisem, J.C. Martinez, B. Stauble-Pumpin and R. Griessen, Nature **399**, 439 (1999).

[2] F.C. Klaassen, G. Doornbos, J.M. Huijbregtse, R.C.F. van der Geest, B. Dam and R. Griessen, Phys. Rev. B **64**, 184523 (2001).

[3] Q. Li, M. Suenaga, Z. Ye, S.R. Foltyn and H. Wang, Appl. Phys. Lett. **84**, 3528 (2004).

[4] B. Maiorov, H. Wang, P.N. Arendt, S.R. Foltyn and L. Civale, Ceramic Transactions **160**, 3 (2005).

[5] L. Civale, B. Maiorov, A. Serquis, J.O. Willis, J.Y. Coulter, H. Wang, Q.X. Jia, P.N. Arendt. J.L. MacManus-Driscoll, M.P. Maley, and S.R. Foltyn, Appl. Phys. Lett. **84**, 2121 (2004).

[6] A.I. Larkin and Yu V. Ovchinnikov, J. Low Temp. Phys. **34**, 409 (1979).

[7] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996) 2nd ed..

[8] J.M. Huijbregtse, B. Dam, R.C.F. van der Geest, F.C. Klaassen, R. Elberse, J.H. Rector and R. Griessen, Phys. Rev. B **62**, 1338 (2000).

[9] D.R. Nelson and V.M. Vinokur, Phys. Rev. B **48**, 13060 (1993).

[10] L. Radzihovsky, Phys. Rev. Lett. **74**, 4923 (1995).

[11] Y. Nonomura and X. Hu, Europhys. Lett. **65**, 533 (2004)

[12] J.P. Rodriguez, Phys. Rev. B **70**, 224507 (2004).

[13] P.H. Kes and C.C. Tsuei, Phys. Rev. B **28**, 5126 (1983).

[14] S.J. Mullock and J.E. Evetts, J. Appl. Phys. **57**, 2588 (1985).

[15] C. Zeng, P.L. Leath, and D.S. Fisher, Phys. Rev. Lett. **82**, 1935 (1999).

[16] T. Nattermann and S. Scheidl, Adv. Phys. **49**, 607 (2000).

[17] M. Chandran, R.T. Scalettar and G.T. Zimanyi, Phys. Rev. B **69**, 024526 (2004).

[18] M. Menghini, Y. Fasano, F. de la Cruz, S.S. Banerjee, Y. Myasoedov, E. Zeldov, C.J. van der Beek, M. Konczykowski, and T. Tamegai, Phys. Rev. Lett. **90**, 147001 (2003).

[19] D. Hull and D.J. Bacon, *Introduction to Dislocations*, 3rd ed. (Pergamon, Oxford, 1984).

[20] E.H. Brandt, J. Low Temp. Phys. **26**, 735 (1977).

[21] A material line defect with an effective circular crossection will begin to trap two vortex lines in the pristine vortex lattice at probabilities $p = \pi r_{\mathrm{p}}^2/(\sqrt{3}/2)a_\triangle^2$ larger than $\pi/2\sqrt{3} \cong 0.907$, where $r_{\mathrm{p}} = a_\triangle/2$.

[22] F. Reif, *Fundamentals of Statistical and Thermal Physics* (McGraw-Hill, Boston, 1965).

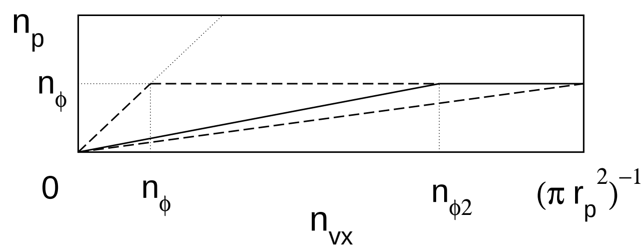

FIG. 1: Sketched is the density of vortex lines pinned to material line defects versus the net density of vortex lines. The scale is assumed to be large compared to the accommodation scale ($B_{\phi 1}$), which is not shown. (See text and ref. [9].)

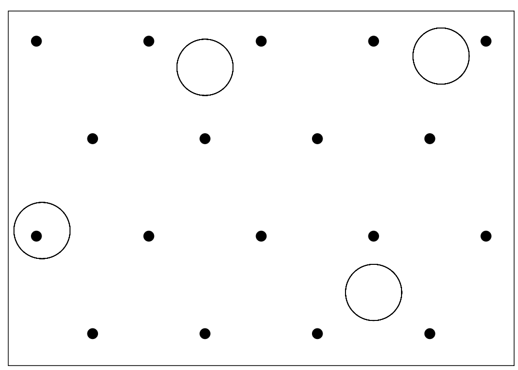

FIG. 2: Sketched is a portion of the vortex lattice (dots) found inside of a Larkin domain, with a random arrangement of linear pinning centers (circles).

[23] L. Civale, A.D. Marwick, T.K. Worthington, M.A. Kirk, J.R. Thompson, L. Krusin-Elbaum, Y. Sun, J.R. Clem, and F. Holtzberg, Phys. Rev. Lett. **67**, 648 (1991).

[24] L. Civale and B. Maiorov, private communications.

[25] A. Diaz, L. Mechin, P. Berghuis and J.E. Evetts, Phys. Rev. Lett. **80**, 3855 (1998).

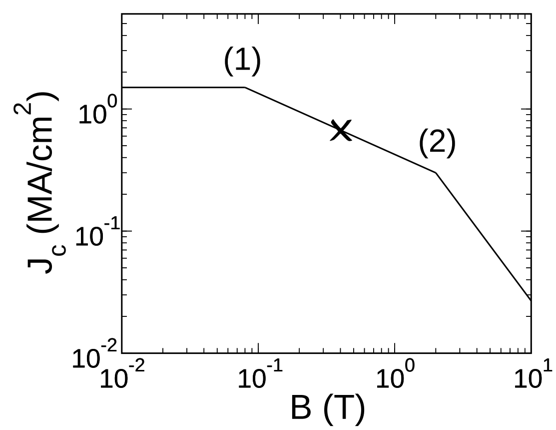

FIG. 3: Sketched is a typical profile of the critical current density measured in films of PLD-YBCO as a function of external magnetic field oriented near the c axis. The "$\times$" marks the power-law regime in magnetic field, $[B_1, B_2]$.

material line defects	d_{p}	$B_{\phi 2}$	α_{cp}
saturated: $n_{\mathrm{p}} = n_{\phi}$	0	–	3/2
[a] row, period D	1	$\Phi_0/(\sqrt{3}/2)D^2$	1
[a] triangular lattice	2	$\Phi_0 \cdot n_{\phi}$	1/2
[b] random, $c_{66}/f_{\mathrm{p}} \to \infty$	2	$\Phi_0/\pi r_{\mathrm{p}}^2$	1/2

[a]at commensuration.

[b]with hard-core repulsion (see text and refs. [2] and [8]).

TABLE I: Listed are various examples of pinning by material line defects that obey the scaling ansatz (5) for the density of pinned vortex lines as a function of magnetic field.

Laboratory	thick	T	$J_c(0+)$	B_{cp}	α	B_ϕ	$\sqrt{B_1 B_2}$	$J_c(\sqrt{B_1 B_2})$	$B_{\phi 2}/B_{\text{cp}}$	B_2/B_1
Amsterdam, ref. [2]	0.1μm	40 K	10 MA/cm^2	8.4 T	0.64	0.07 T	0.78 T	2.3 MA/cm^2	0.1	6.7
LANL, ref. [4]	4.3μm	75.5 K	1.0 MA/cm^2	0.08 T	0.46	[a] 0.17 T	0.31 T	0.4 MA/cm^2	2.9	20
BNL, ref. [3]	3μm	77 K	1.3 MA/cm^2	0.14 T	0.67	[a] 0.17 T	0.22 T	0.3 MA/cm^2	4.7	20

[a] Rough estimate taken from ref. [8].

TABLE II: Listed above are theoretical predictions, $[B_{\text{cp}}, B_{\phi 2}]$, versus experiment, $[B_1, B_2]$. (See Fig. 3.) The effect of point pinning is neglected ($j_c' = 0$).